Supplementary File

This supplementary document provides the formulae used for the various area under the curve calculation options available in the Time Series Response Analyser. These are all based on the trapezium rule used for basic mathematical integration.

Table 1. The variables used throughout this document to explain the area under the curve calculations.

Variable	Definition
N	Total number of time-points (including first measured value)
A_n	Area of nth segment
t_n	Time at n^{th} time-point
t_{n-1}	Time at previous time-point
$f[t_n]$	Value measured at t_n
$f[t_{n-1}]$	Value measured at t_{n-1}
y	Threshold value

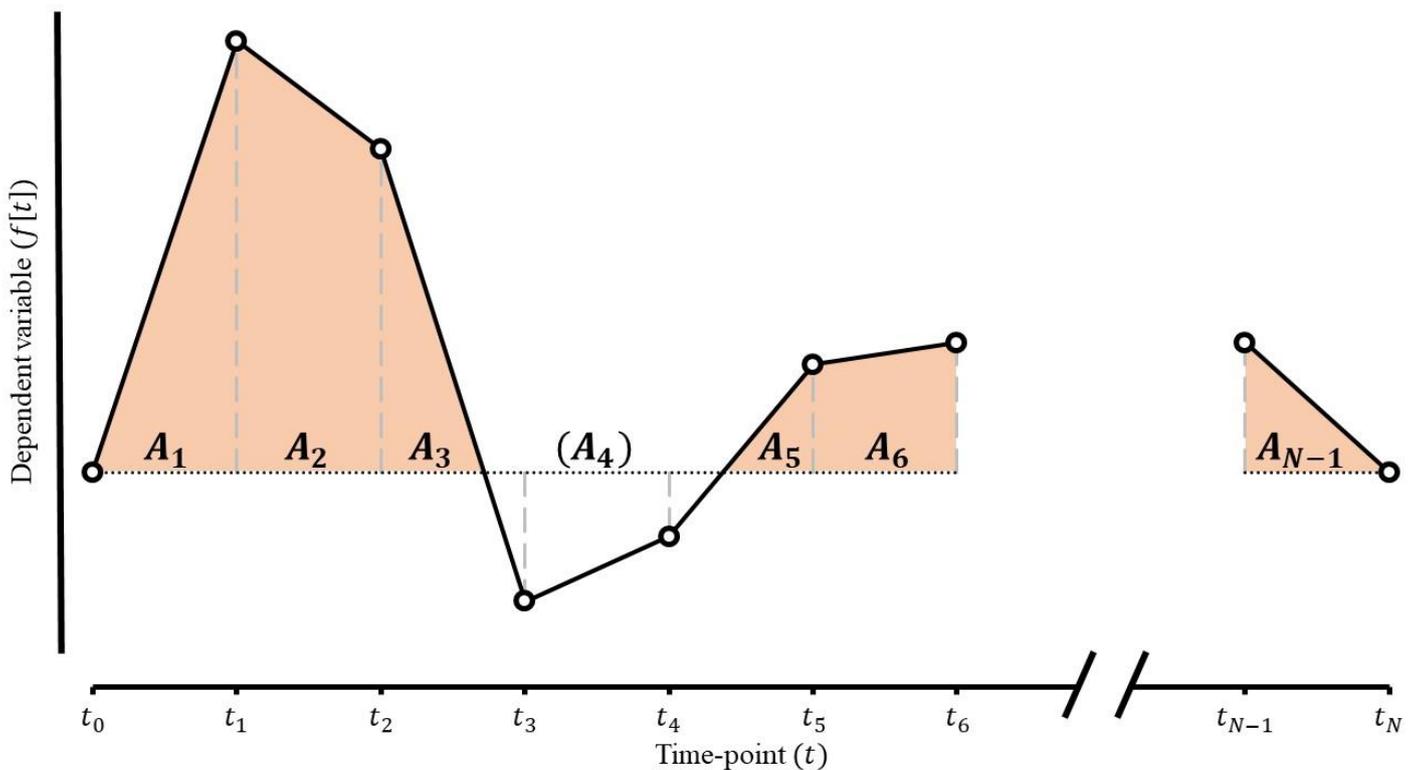*Note:* a response characterised by N time-points will have $N-1$ segmental areas

Figure 1. A graphical representation of a complete time-series response consisting of N time-points and $N-1$ segments. Areas shaded red each contribute to the overall calculation.

Table 2. The threshold value (y) defined under each calculation option.

Calculation	Threshold value (y)
tAUC	$f[t] = 0$
iAUC	$f[t] = $ baseline (first measured value)
iAUCmin	$f[t] = $ nadir (minimum measured value)
iAUCx	$f[t] = $ user-defined absolute value

iAUC, incremental area under the curve; iAUCmin, incremental area under the curve relative to nadir; iAUCx, incremental area under the curve relative to arbitrary fixed threshold; tAUC, total area under the curve.

Note: if calculation option ends in 'cut', segmental calculations are terminated when the response line crosses the threshold value (positive to negative) for the first time. The positive area from the final segment is retained.
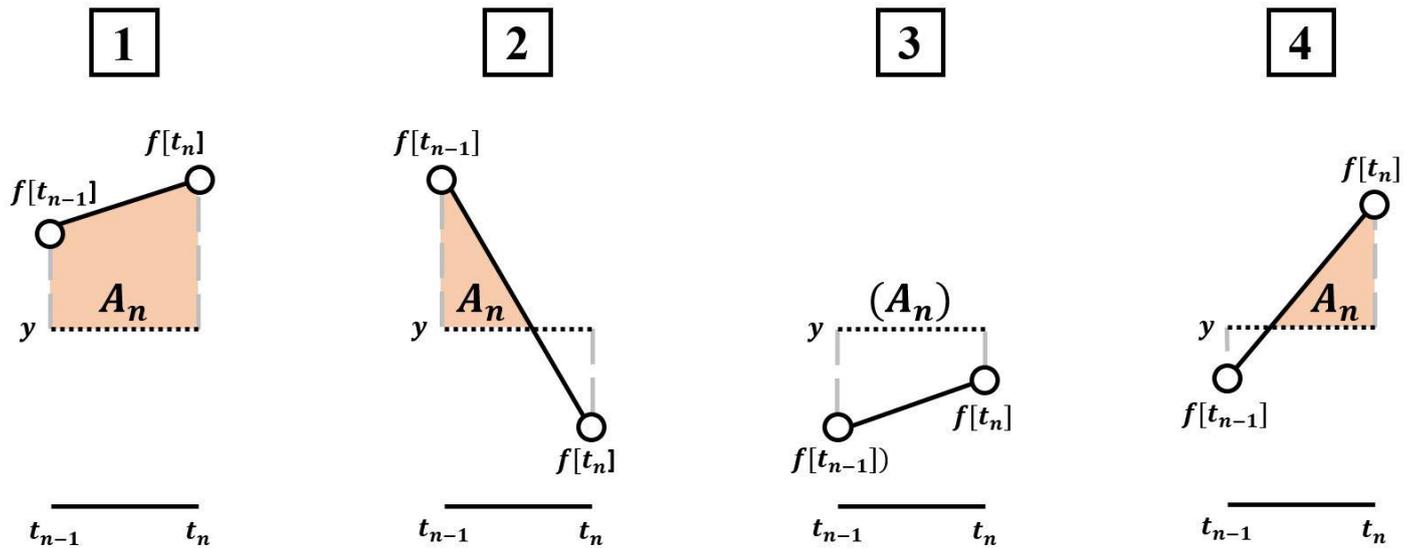
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Figure 2. The four possible scenarios for each pair of data-points, depending on their relation to the threshold value.

Table 3. The equations for the segmental areas under each of the four scenarios.

Scenario	Equation
1	$A_n = \dfrac{(f[t_n] + f[t_{n-1}])(t_n - t_{n-1})}{2} - y(t_n - t_{n-1})$
2	$A_n = \dfrac{(f[t_{n-1}] - y)}{(f[t_{n-1}] - f[t_n])} \times \dfrac{(f[t_{n-1}] - y)(t_n - t_{n-1})}{2}$
3	$A_n = 0$
4	$A_n = \dfrac{(f[t_n] - y)}{(f[t_n] - f[t_{n-1}])} \times \dfrac{(f[t_n] - y)(t_n - t_{n-1})}{2}$

$$Total\ calculated\ area = \sum_{n=1}^{N-1} A_n$$

Note: the Time Series Response Analyser identifies the scenario for each pair of data-points and applies the appropriate equation from this table. The segmental areas displayed in the 'Tabulated Data' worksheet are the result of each individual calculation, and the final area under the curve value is the sum of all segments within a given trial.

Reference

Narang, B. J., Atkinson, G., Gonzalez, J. T., & Betts, J. A. (2020). A tool to explore discrete-time data: the time series response analyser. *International Journal of Sport Nutrition and Exercise Metabolism, TBC.*

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